Lake Zurich High School

Getting Started Packet for Honors Pre-Calculus

Welcome to Honors Pre-Calculus.

Honors Pre-Calculus will refresh your Algebra skills, review polynomial functions and graphs, explore trigonometry in depth, and give you a brief introduction to topics in Calculus. The topics in the course can be abstract but they have practical application and serve as a foundation for further study in Calculus and other college level mathematics.

This Summer Review Packet was prepared for students to be able to review material from Algebra II and Advanced Math before beginning the school year. The packet is to be completed by August 28, 2009 (the first Friday of the new school year). We thought you might want to get a head start on it over the summer. You will hand in the completed packet on August 28, 2009 and it will be graded based upon completion and correctness. You **MUST** show all work in order to receive credit! You will also be quizzed on the material that same day.

Calendar of first week of the 2009-2010 school year.

Tuesday, August 25	Half Day → class handouts and syllabus
Wednesday, August 26	First Full Day → get books & begin Chapter P
Friday, August 28	Summer Review Packet DUE and Quiz on material

A "Getting Started Packet Resource Companion" is available on-line on the LZHS Mathematics Department Homepage:

http://www.lz95.org/lzhs/Math/mathhome.htm

This Resource Companion has some Algebra examples that may be helpful if you have forgotten a particular technique or concept. Although the Resource Companion does not contain examples for every type of question in the Getting Started Packet, it can be helpful for many of the questions.

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Pre-Calculus Summer Review Guide

1.) Geometry Topics

Equations of a line:

1. Slope intercept: $y = mx + b$

   where $slope = \frac{y_2 - y_1}{x_2 - x_1}$
2. Point slope: $y - y_1 = m(x - x_1)$
3. Standard: $Ax + By + C = 0$

Directions: Solve each problem in the space provided, circling your final answer. Put all answers in standard form.

1. Write the equation of the line parallel to $2x - 6y = -1$ and containing the x-intercept of $4x - 3y = 12$.

2. Write the equation of the line in slope intercept form through the point with coordinates (-4,6) and perpendicular to $3x - 2y = 8$.

3. Find the value of "a" if a line containing the point (a, -2a) has a y-intercept of 6 and slope of -2/3.

4. Write the equation of the perpendicular bisector of the segment joining the points with coordinates of (-3,4) and (5,-2).

2.) Rules of Exponents

Properties:

$a^m \cdot a^n = a^{m+n}$ $\quad$ $(a^m)^n = a^{mn}$ $\quad$ $a^{\frac{p}{r}} = \sqrt[r]{a^p}$

$a^{-n} = \frac{1}{a^n}$ $\quad$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\quad$ $\frac{a^m}{a^n} = a^{m-n}$

Directions: Simplify each in the space provided, showing all steps. Answers should have positive exponents. Circle your answer.

1. $(2x^2y)^0(3xy)$

2. $a^{-2}b^3a^3$

3. $\frac{4^{-5}4^6}{4^2}$

4. $(2x)^{-2}(2y)^3(4x)$

5. $\frac{\left(a^{-3}b^2c\right)^{-2}}{\left(ab^{-2}c^3\right)^{-1}}$

6. $\frac{2^4 8^3 16^{-2}}{32^{-1}}$

7. $\left(\frac{5u^2v}{2uv^2}\right)^2\left(\frac{-3uv}{2u^2v}\right)^3$

8. $(3^{-1} + 2^{-1})^2$

9. $\frac{a^{-1} - 3a^{-2}}{2^{-2}}$

3.) Factoring

Strategies to use:
1. Greatest Common Factor (GCF)
2. Difference of Squares
3. Trinomials
4. Sum and Difference of Cubes
   $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
5. Grouping

Directions: Factor each of the following completely, circling your final answer.

1. $-20a^2 + 5t^2$
2. $3x^3 – 375$
3. $2x^3 + 6x^2 + 4x + 12$
4. $x^6 – 64$
5. $7x^2 + 23x + 6$
6. $63w^2 – 46wv + 8v^2$
7. $48x^2 – 119\,mx – 24m^2$
8. $10x^2 + 29x - 21$

4.) Function Notation

Directions: Find the value of each in the space provided, showing all steps. Circle your answer.

Given: $f(x) = 3x - 7$ $g(x) = x^2 + 3$

1. Find f(-1)
2. f (x + 3)
3. f(f(x))

4. g(x + 2) – g(x)
5. f(g(2))
6. g(f(2))

5.) Properties of Logarithms

DEFINITION	$\log_a x = y \quad \leftrightarrow \quad a^y = x$
EXPONENTS	$\log_b m^n = n \cdot \log_b m$
QUOTIENT	$\log_b m - \log_b n = \log_b \frac{m}{n}$
PRODUCT	$\log_b m + \log_b n = \log_b m \cdot n$

Solve the following logarithmic equations:

1. $\log_3 x^3 = 9$
2. $\log_{\frac{1}{2}} \frac{1}{16} = x^2$
3. $\log_2 15 + \log_2 14 - \log_2 105 = \log_2 x$

4. $\log_2 x = 12$
5. $\log_5 125 = x$
6. $3\log_2 x = 12$

6.) Division of Polynomials

ex1) Use synthetic division to find the remainder of $\frac{2x^2+3x-5}{x+6}$

-6 | 2 3 -5

2

-6 | 2 3 -5

-12

2

-6 | 2 3 -5

-12

2 -9

-6 | 2 3 -5

-12 54

2 -9

-6 | 2 3 -5

-12 54

2 -9 **49**

49 is the remainder

ex2) Find $\frac{12x^2+8x-15}{6x-5}$ using long division

$$6x-5 \overline{)\ 12x^2 + 8x - 15}$$

$$2x$$

$$-(12x^2 - 10x)$$

$$18x - 15$$

$$6x-5 \overline{)\ 12x^2 + 8x - 15}$$

$$2x + 3$$

$$-(12x^2 - 10x)$$

$$18x - 15$$

$$-(18x - 15)$$

$$0$$

$2x + 3$ is the answer

ex3) Simplify $\frac{5x^2 - 5x - 60}{x^2 - 2x - 8}$.

$$\frac{5(x^2 - x - 12)}{x^2 - 2x - 8}$$

$$\frac{5(x - 4)(x + 3)}{(x - 4)(x + 2)}$$

$$\frac{5(x + 3)}{(x + 2)}$$

Solve the following problems involving division of polynomials:

1. What is the remainder when $x^3 - 6x^2 - 5x - 7$ is divided by (x – 5) using synthetic division?

2. Find $\frac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1}$ by long division.

3. 1. Simplify $\frac{x^2 - 4x - 21}{x + 3}$ by factoring.

7.) Rational Expressions

Simplify the following expressions :

1. $\frac{x}{xy^2} - \frac{2x}{x^2}$

2. $\frac{x}{x-3} + \frac{2}{3x+4}$

3. $\frac{x^2 - 5x + 6}{x - 2}$

4. $\frac{1-x}{x-1}$

5. $\frac{2x^2 + x - 6}{x^2 + 4x - 5} \bullet \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x}$

8.) Radicals

Simplify radicals whenever possible:

ex1) $\sqrt{x^5 y^7 z^6}$

$\sqrt{x^2\, x^2\, x \cdot y^2\, y^2\, y^2\, y \cdot z^2\, z^2\, z^2}$

$x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z\sqrt{x \bullet y}$

$x^2 y^3 z^3 \sqrt{xy}$

ex2) $\sqrt[3]{-32}$

$\sqrt[3]{-2 \cdot -2 \cdot -2 \cdot 2 \cdot 2}$

$-2\sqrt[3]{4}$

ex3) $3x\sqrt{2x} \bullet 4x^2\sqrt{5x}$

$3x \cdot 4x^2\sqrt{2x \cdot 5x}$

$12x^3\sqrt{10x^2}$

$12x^4\sqrt{10}$

ex4) $5\sqrt{27} + 4\sqrt{2} + 7\sqrt{3}$

$15\sqrt{3} + 4\sqrt{2} + 7\sqrt{3}$

$22\sqrt{3} + 4\sqrt{2}$

Rationalize fractions with radicals in the denominator:

ex5) $\sqrt{\frac{24}{15}}$

$\sqrt{\frac{8}{5}}$

$\frac{2\sqrt{2}}{\sqrt{5}}$

$\frac{2\sqrt{2}}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{2\sqrt{10}}{5}$

ex6) $\frac{2}{\sqrt[4]{32}}$

$\frac{2}{\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}}$

$\frac{\cancel{2}}{\cancel{2} \cdot \sqrt[4]{2}}$

$\frac{1}{\sqrt[4]{2}}$, now rationalize

$\frac{1}{\sqrt[4]{2}} \bullet \frac{\sqrt[4]{2 \cdot 2 \cdot 2}}{\sqrt[4]{2 \cdot 2 \cdot 2}}$

$\frac{\sqrt[4]{8}}{\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}}$

$\frac{\sqrt[4]{8}}{2}$

ex7) $\frac{1}{2-\sqrt{3}}$

$\frac{1}{2-\sqrt{3}} \bullet \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\frac{2+\sqrt{3}}{2^2 - \left(\sqrt{3}\right)^2}$

$\frac{2+\sqrt{3}}{4-3}$

$2+\sqrt{3}$

Simplify the following:

1. $\sqrt[3]{24}$
2. $\sqrt[3]{-40x^6 y^7}$
3. $\sqrt{75x^3} \bullet \sqrt{5x^3}$
4. $2\sqrt{48} - 3\sqrt{27}$

Rationalize the following:

5. $\frac{5}{2\sqrt{3}}$
6. $\frac{2}{\sqrt[3]{5}}$
7. $\frac{2}{3+\sqrt{7}}$

True or False – Explain why.

8. $\sqrt{x} = \left|\sqrt{x}\right|$ for all x
9. $\sqrt{a^2 + b^2} = a + b$
10. $\sqrt[4]{16} = \sqrt[4]{-16}$

9.) Simplify Complex Fractions

When simplifying complex fractions, multiply both the numerator and denominator by the reciprocal of the denominator. Remember to also look for common factors to simplify.

1. $\dfrac{\dfrac{x^2}{x-1}}{\dfrac{2x}{x-1}}$

2. $\dfrac{\dfrac{3}{x+1}-4}{\dfrac{2x}{x+1}}$

3. $\dfrac{\dfrac{x^2+2x+1}{x^2-4}}{\dfrac{x+1}{x^2-x-6}}$

10.) Basic Polynomial Graphs -- Translations/Transformations of Parent Functions

Be able to quickly sketch these five basic "parent" graphs from memory:

$y=\sqrt{x}$ $y=x^2$ $y=|x|$ $y=x^3$ $y=\frac{1}{x}$

To translate or move a parent function, use these basic examples:

the equation $y=(x-5)^2-3$ moves the graph 5 units to the right and 3 units down

the equation $y=(x+2)^2+4$ moves the graph 2 units to the left and 4 units up

To transform or stretch/shrink a parent function, use these basic examples:

the equation $y=\frac{1}{3}x^2$ shrinks the graph vertically so the height is $\frac{1}{3}$ as tall as the parent graph.

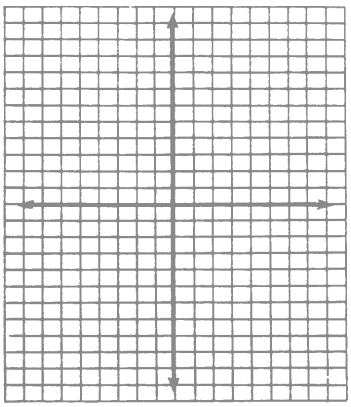
the equation $y=4x^2$ stretches the graph vertically so the height is 4 times as tall as the parent graph.

the equation $y=\left(\frac{1}{2}x\right)^3$ stretches the graph horizontally so it is 2 times as wide as the parent graph.

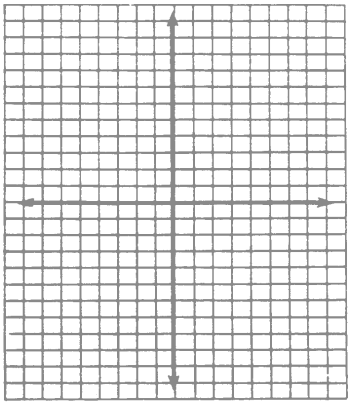
the equation $y=(3x)^3$ shrinks the graph horizontally it is $\frac{1}{3}$ as wide as the parent graph.

Use the equations below to ***ACCURATELY*** sketch the graphs the functions without the aid of your calculator. **Please label at least 3 points, including vertices, and also label the asymptotes if necessary.**

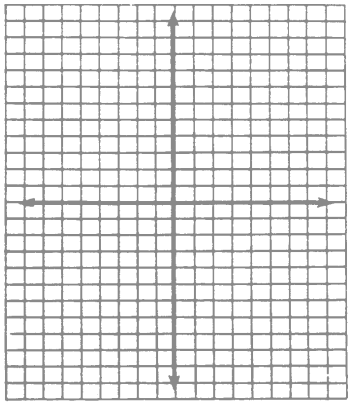
1. $y=|x-3|+2$



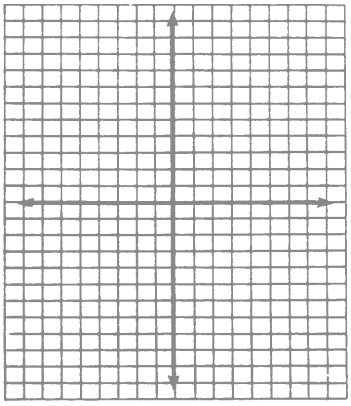
2. $y=\sqrt{x+2}$



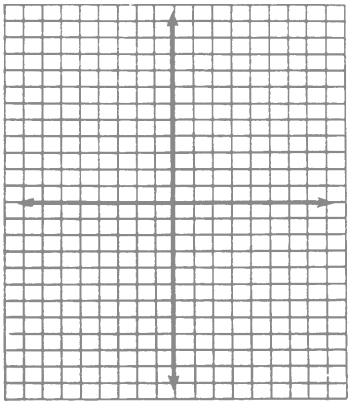
3. $y=\frac{1}{x+2}-2$



4. $y=\frac{1}{2}x^2$



5. $y=3(x-1)^3$



ANSWERS:

Page 1: Geometry Topics

1. $x-3y-3=0$
2. $y=-\frac{2}{3}x+\frac{10}{3}$
3. $-\frac{9}{2}$
4. $y=\frac{4}{3}x-\frac{1}{3}$ or $4x-3y-1=0$

Page 2: Rules of Exponents

1. $3xy$
2. ab^3
3. $\frac{1}{4}$
4. $\frac{8y^3}{x}$
5. $\frac{a^7c}{b^6}$
6. 2^{10}
7. $-\frac{675}{32uv^2}$
8. $\frac{25}{36}$
9. $\frac{4(a-3)}{a^2}$ or $\frac{4a-12}{a^2}$

Page 3: Factoring

1. $5(t+2a)(t-2a)$ or equivalent
2. $3(x-5)(x^2+5x+25)$
3. $2(x+3)(x^2+2)$
4. $(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$
5. $(7x+2)(x+3)$
6. $(9w-4v)(7w-2v)$
7. $(3x-8m)(16x+3m)$
8. $(5x-3)(2x+7)$

Page 4: Function Notation

1. –10
2. $3x+2$
3. $9x-28$
4. $4x+4$
5. 14
6. 4

Page 5: Properties of Logarithms

1. 27
2. ±2
3. 2
4. 4096
5. 3
6. 16

Page 6: Division of Polynomials

1. –57
2. $5x+2$
3. $x-7$

Page 7: Rational Expressions

1. $\frac{x-2y^2}{xy^2}$
2. $\frac{3(x^2+2x-2)}{(x-3)(3x+4)}$
3. $x-3$
4. –1
5. $\frac{(x+2)(x-2)}{2(x+5)}$

Page 8: Radicals

1. $2\sqrt[3]{3}$
2. $-2x^2y^2\sqrt[3]{5y}$
3. $5x^3\sqrt{15}$
4. $-\sqrt{3}$
5. $\frac{5\sqrt{3}}{6}$
6. $\frac{2\sqrt[3]{25}}{5}$
7. $3-\sqrt{7}$
8. True
9. False, just let a=3 and b=4 for instance
10. False

Page 9: Simplify Complex Fractions

1. $\frac{x}{2}$
2. $\frac{-4x-1}{2x}$ or $\frac{-(4x+1)}{2x}$
3. $\frac{(x+1)(x-3)}{(x-2)}$

Page 10: Basic Polynomials Graphs

(use graphing calculator to verify graphs)